Inequality
https://www.linkedin.com/groups/8313943/8313943-6379947029581434881
Prove the inequality

$$\frac{x_1}{1+x_1^2}+\frac{x_2}{1+x_1^2+x_2^2}+\ldots+\frac{x_n}{1+x_1^2+x_2^2+\ldots+x_n^2}<\sqrt{n},\ \forall x_1,x_2,\ldots,x_n\in\mathbb{R}.$$

Solution by Arkady Alt, **San Jose**,**California**, **USA**.
Let $a_k:=\dfrac{x_k}{1+\sum_{i=1}^k x_i^2}$, $b_k:=\dfrac{1}{1+\sum_{i=1}^k x_i^2}$, $k=1,2,\ldots,n$ and $S_n:=\sum_{k=1}^n a_k$.
Since and by Cauchy Inequality (or, by QM-AM Inequality) $S_n\le\sqrt{n}\cdot\sqrt{\sum_{k=1}^n a_k^2}$
and $a_k^2=\dfrac{x_k^2}{\left(1+\sum_{i=1}^k x_i^2\right)^2}\le\dfrac{x_k^2}{\left(1+\sum_{i=1}^k x_i^2\right)\left(1+\sum_{i=1}^{k-1} x_i^2\right)}=b_{k-1}-b_k, k=1,2,3,\ldots,n$
(here we set $b_0:=1$) then $S_n\le\sqrt{n}\cdot\sqrt{a_1^2+\sum_{k=2}^n a_k^2}\le\sqrt{n}\sqrt{1-b_n}<\sqrt{n}$.